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LETTER TO THE EDITOR

On Monte Carlo generation and study of anisotropy of lattice animals

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Abstract. A new type of cluster in the universality class of lattice animals is introduced based on a stochastic form of the Martin algorithm by the Monte Carlo method. Lattice animals with sizes up to 1000 sites are generated. The asymptotic radius of the gyration exponent is consistent with the value 0.64, corresponding to a fractal dimension of 1.56. An investigation of anisotropy of these animals shows that small animals are anisotropic while larger ones are not.

Lattice animals are much studied objects in connection with clustering and nucleation [1-3] and in percolation theory [4-6]. Statistics and properties of animals have been investigated by means of exact enumeration [7, 8], Monte Carlo methods [4, 9-12], momentum space renormalisation group [13] and phenomenological renormalisation group [14]. Recently Duarte [15] studied lattice trees which are believed to share the same universality class of lattice animals using a Monte Carlo method similar to that of [4]. It is difficult to generate large lattice animals directly using the Monte Carlo method. If one generates percolation clusters at a constant value of the site occupancy p with p less than the percolation threshold p_c , then those clusters with characteristic linear dimension $R \gg \xi(p)$ where $\xi(p)$ is the correlation length will be self-similar with the fractal dimension of lattice animals. However, since the probability of finding an N -site cluster with $R \gg \xi(p)$ decreases exponentially with N [5, 6], it is difficult to generate lattice animals with large N . Using a method similar to that of the enrichment model for linear polymers, Havlin *et al* [12] generated lattice animals of size about 300. In this letter we wish to introduce a new method for generating directly large lattice animals by the Monte Carlo method. It is a stochastic form of the Martin algorithm [8, 16]. A similar version of it had already been used to produce self-avoiding walk configurations with satisfactory results [17]. It is simply implemented by taking the FORTRAN program given in [8] and changing statement 6 to

```
if(iocc(inow).eq.0.and.ranf(0).lt.p)goto 3
```

Here `ranf(0)` generates a real random number between zero and one and the fixed parameter p is a real number between zero and one. In addition, since in the Monte Carlo method, not all the neighbouring sites of the lattice animal are investigated, the information on which neighbouring sites had been investigated should be stored for each site of the animal. Clearly for $p = 1$ this reduces to the exact enumeration program of lattice animals. We have calculated the radius of gyration of clusters generated this

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way for $p = 0.2, 0.3$ and 0.4 . For each p , a fixed number of configurations N_c of lattice animals of size N are generated and their radii of gyration measured and averaged using one set of random numbers. The whole process is then repeated using different sets of random numbers. Each set of random numbers is called one trial. The radii of gyration obtained in different trials are then averaged over the total number of trials N_T . Since each trial is an independent event in contrast to the different configurations of lattice animals produced in the same trial, we can estimate the error by calculating the fluctuations in the averaged radius of gyration over the N_T trials. We have generated lattice animals with sizes up to $N = 1000$, with $N_c = 4000$ and N_T up to 500. The values of the radii of gyration differ for different p , but the exponent obtained by taking the slope of the $\log R_g$ against $\log N$ plot is consistent with the value of 0.64 for large N , the value for lattice animals in two dimensions [14]. In figure 1 we give the $\log R_g$ against $\log N$ plot for the three values of p . The result for the exponent defined through $R_g \sim N^\nu$, where R_g is the radius of gyration, is shown in figure 2. The values of the exponent are obtained by taking successive slopes in the log-log plot of R_g against N .

Recently there has been a lot of interest in the study of anisotropy in random fractal clusters [18, 19]. Family *et al* [18] studied the anisotropy of lattice animals on the square lattice using the exact enumeration method. They diagonalised the radius of gyration tensor R_{ij}^2 and calculated the principal radii of gyration. The anisotropy A_N of an N -site cluster is then defined to be the ratio $R_{N,\min}^2/R_{N,\max}^2$, where $R_{N,\min}^2$ and $R_{N,\max}^2$ are the smaller and the larger eigenvalues of the radius of gyration tensor, respectively. But since this ratio is calculated for each particular configuration of the animal and then the average of these ratios is taken over the different configurations, this only measures on average how anisotropic any particular lattice animal can be.

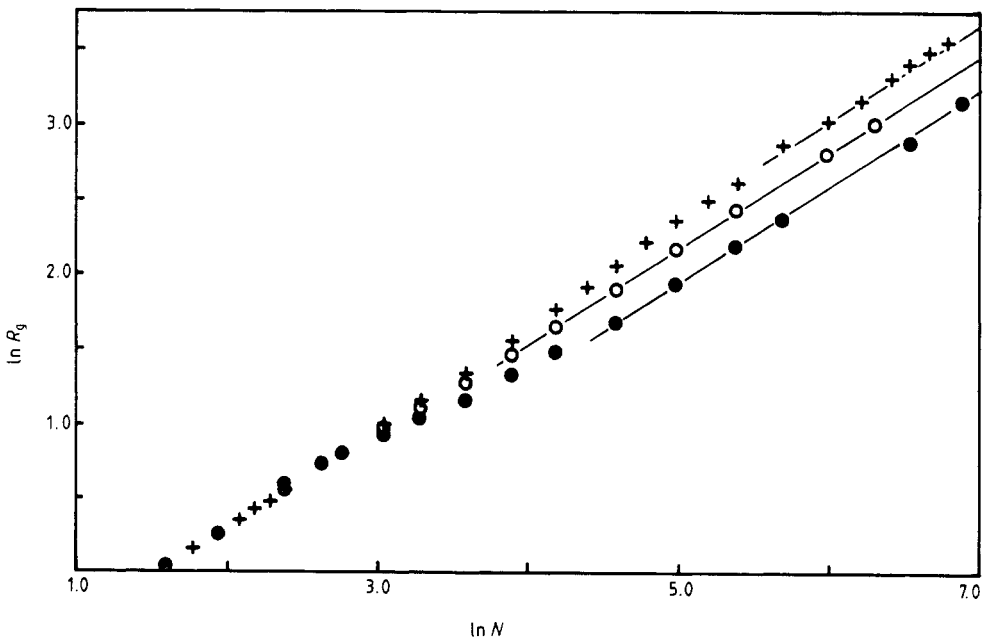


Figure 1. Log-log plot of R_g against N for $p = 0.2$ (+), 0.3 (O) and 0.4 (●). Asymptotic slopes of all three curves give an exponent compatible with 0.64

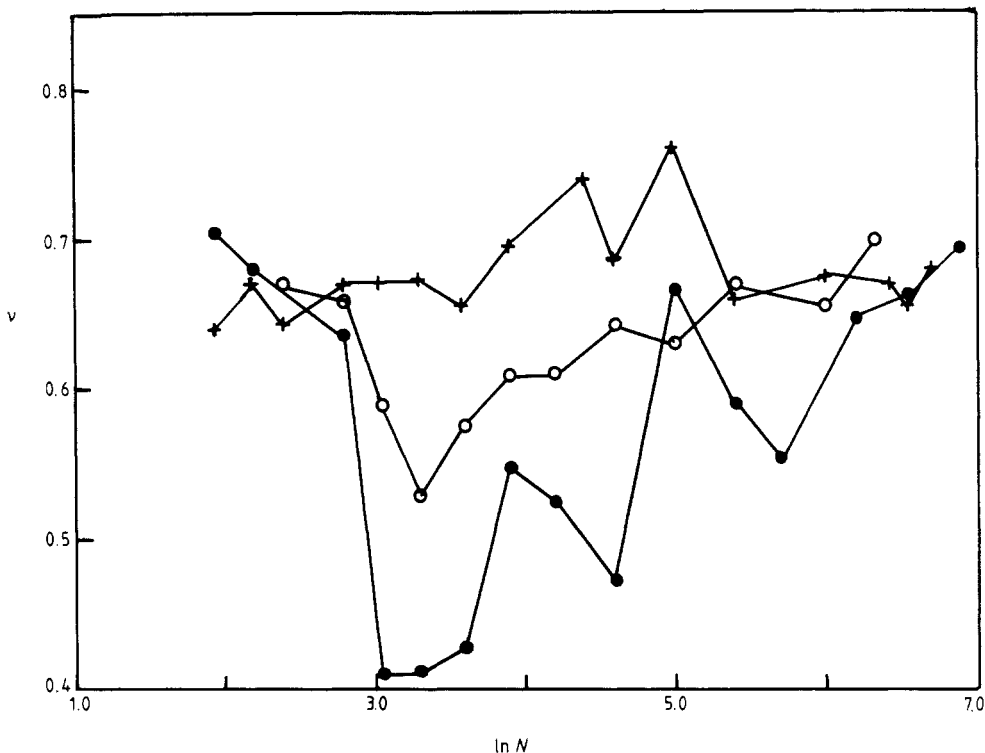


Figure 2. Value of the exponent obtained by taking successive slope of the curves in figure 1, plotted against $\log N$, for $p = 0.2$ (+), 0.3 (O) and 0.4 (●).

It does not measure the anisotropy of lattice animals in the sense of whether or not lattice animals are anisotropic when averaged over all configurations. Here we measure anisotropy in a different way [5]. We choose the origin of the coordinate system at the centre of mass of the animal with X and Y directions the same as in the original square lattice. We define axis sites of the animal as sites with centre of mass coordinates x_i, y_i such that $|x_i|$ or $|y_i|$ is less than 0.5 and diagonal sites as sites such that $|x_i - y_i|$ or $|x_i + y_i|$ is less than 0.5 . (We have taken unit lattice spacing.) We calculate separately the quantities $\langle R_X^2 \rangle = \langle N_X^{-1} \sum (x_i^2 + y_i^2) \rangle$ and $\langle R_D^2 \rangle = \langle N_D^{-1} \sum (x_i^2 + y_i^2) \rangle$ in which the summation is over the N_X axis and N_D diagonal sites respectively and we average them over the animal configurations. The difference between $\langle R_X^2 \rangle$ and $\langle R_D^2 \rangle$ is a measure of the anisotropy. In table 1 we show the values $\langle R_X^2 \rangle^{1/2}$ and $\langle R_D^2 \rangle^{1/2}$ and the ratios $\langle R_X^2 \rangle^{1/2} / \langle R^2 \rangle^{1/2}$ and $\langle R_D^2 \rangle^{1/2} / \langle R^2 \rangle^{1/2}$ where $\langle R^2 \rangle^{1/2}$ is the average radius of gyration, for different animal sizes N . It is seen that these ratios are always less than one. In a circle of radius R on a two-dimensional lattice with all sites occupied, the density of sites on the plane and the density of sites on the axis are constant. We have $\langle R^2 \rangle = \frac{1}{2} R^2$ and $\langle R_X^2 \rangle = \frac{1}{3} R^2$. This gives $\langle R_X^2 \rangle / \langle R^2 \rangle = \langle R_D^2 \rangle / \langle R^2 \rangle = \frac{2}{3}$. For lattice animals, neither the sites on the plane nor on the axis or diagonals have constant density. We therefore do not know *a priori* what value these ratios should approach in case of isotropy, except that they should approach the same value less than unity. This seems to be the case here for large N . But for N less than about 500 there is definite evidence of anisotropy in lattice animals.

Table 1. $\langle R^2 \rangle^{1/2}$, $\langle R_x^2 \rangle^{1/2}$, $\langle R_D^2 \rangle$.

N	$\langle R^2 \rangle^{1/2}$	$\langle R_D^2 \rangle^{1/2}$	$\langle R_x^2 \rangle^{1/2}$
$p = 0.3$			
50	4.3242 ± 0.0205	3.1064 ± 0.0197	3.1488 ± 0.0207
100	6.6632 ± 0.0507	4.5552 ± 0.0595	4.8463 ± 0.0600
200	10.5036 ± 0.1450	7.1473 ± 0.1524	7.5475 ± 0.1654
500	19.3323 ± 0.4469	13.3197 ± 0.5507	12.3208 ± 0.4196
600	21.7447 ± 0.5862	14.3716 ± 0.6874	14.3707 ± 0.6014
700	25.4910 ± 0.7794	16.4477 ± 1.0043	16.9405 ± 0.7720
800	27.3258 ± 0.8751	18.0137 ± 0.9100	17.0580 ± 0.9291
$p = 0.4$			
10	1.6925 ± 0.0025	1.2744 ± 0.0023	1.6591 ± 0.0024
100	5.5476 ± 0.0304	3.9349 ± 0.0297	4.3062 ± 0.0326
500	14.4695 ± 0.0988	9.7216 ± 0.1029	10.5950 ± 0.1141
700	18.0875 ± 0.1580	12.5087 ± 0.1735	12.6670 ± 0.1670
1000	22.9622 ± 0.0628	15.5094 ± 0.0480	15.7670 ± 0.1504

In order to make sure that lattice animals are indeed anisotropic at small N , we have calculated the measure of anisotropy as defined above by exact enumeration (i.e. with $p = 1$) up to $N = 13$. The result is shown in table 2. We see that up to these sizes, lattice animals on the square lattice are indeed anisotropic.

Table 2. Same as table 1 but with exact enumeration.

N	$\langle R^2 \rangle^{1/2}$	$\langle R_D^2 \rangle^{1/2}$	$\langle R_x^2 \rangle^{1/2}$
3	0.7201	0.5443	0.7201
4	0.9105	0.6977	0.9810
5	1.0809	0.8777	0.9694
6	1.2383	0.9611	1.1027
7	1.3854	1.0438	1.1823
8	1.5243	1.1426	1.2923
9	1.6565	1.2333	1.3633
10	1.7833	1.3299	1.4552
11	1.9031	1.4143	1.5326
12	2.0232	1.5037	1.6200
13	2.1376	1.5819	1.6940

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